Erratum — Colloquium: Mechanical formalisms for tissue dynamics

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In Appendix D.2.2, entitled "Constitutive equations", we have found a technical error concerning Eqs. (D.37,D.39): a rotation term had been forgotten and the validity of these equations was erroneously not mentioned to be restricted to commuting tensors. This error has no consequence on the general validity of the Appendix, although it slightly alters the resolution scheme. As a consequence, Appendix D.2.2 is slightly amended as follows:

Modified Appendix D.2.2

In the small deformation expressions (4) or (8) of the dissipation function, the notation $\dot{\varepsilon}$ or $\dot{\varepsilon}_1$ in fact designates the symmetrised velocity gradient D and its effective counterpart D_1 (see Appendix D.1.4). Although this confusion has no consequence when the deformations ε or ε_1 are small, see Eqs. (D.23,D.24), at large deformations it is necessary to express the dissipation function with respect to the correct kinematic variables:

$$\mathcal{D} = \mathcal{D}\left(D, D_1, \ldots, D_m\right) \tag{4b}$$

$$\mathcal{D} = \mathcal{D}\left(\operatorname{tr} D, \operatorname{tr} D_1, \ldots, \operatorname{dev} D, \operatorname{dev} D_1, \ldots\right) \tag{8b}$$

while the static variables ε, ε_1 are still correct variables for the energy function.

The differentiation rule of the energy and dissipation function given by Eqs. (B.10-B.13) is now rewritten using D and D_1:

$$\operatorname{dev}\sigma \;=\; \frac{\partial \mathcal{D}}{\partial \operatorname{dev} D} + \frac{\partial \mathcal{E}}{\partial \widehat{\operatorname{dev}\varepsilon}} \tag{D.40}$$

$$\operatorname{tr}\sigma \;=\; \frac{\partial \mathcal{D}}{\partial \operatorname{tr} D} + \frac{\partial \mathcal{E}}{\partial \widehat{\operatorname{tr}\varepsilon}} \tag{D.41}$$

$$0 \;=\; \frac{\partial \mathcal{D}}{\partial \operatorname{dev} D_1} + \frac{\partial \mathcal{E}}{\partial \widehat{\operatorname{dev}\varepsilon_1}} \tag{D.42}$$

$$0 \;=\; \frac{\partial \mathcal{D}}{\partial \operatorname{tr} D_1} + \frac{\partial \mathcal{E}}{\partial \widehat{\operatorname{tr}\varepsilon_1}} \tag{D.43}$$

In Eqs. (8b,D.40-D.43), $\widehat{\operatorname{tr}\varepsilon}$, $\widehat{\operatorname{dev}\varepsilon}$, $\widehat{\operatorname{tr}\varepsilon_1}$ and $\widehat{\operatorname{dev}\varepsilon_1}$ are given by Eqs. (D.29,D.30). These equations are solved together with Eq. (14) and yield directly the velocity field $\vec{v}$ (and its symmetrized gradient D), the effective symmetrized velocity gradient D_1 and the stress σ. The evolution of the deformations ε and ε_1 and of the mass density ρ is then obtained from Eqs. (D.23, D.24, 22).

Note that as can be shown from Eqs. (D.23,D.24,D.29):

$$(\partial_t + \vec{v}\cdot\nabla)\left[\widehat{\operatorname{tr}\varepsilon}\right] \;=\; \operatorname{tr} D \tag{D.36}$$

$$(\partial_t + \vec{v}\cdot\nabla)\left[\widehat{\operatorname{tr}\varepsilon_1}\right] \;=\; \operatorname{tr} D_1 \tag{D.38}$$

In highly symmetric geometries such that the symmetrized velocity gradients D and D_1 and the deformations ε and ε_1 remain aligned, one can also show, using Eqs. (D.23,D.24,D.30), that:

$$(\partial_t + \vec{v}\cdot\nabla)\left[\widehat{\operatorname{dev}\varepsilon}\right] \;=\; \operatorname{dev} D$$
$$+\Omega\,\widehat{\operatorname{dev}\varepsilon} - \widehat{\operatorname{dev}\varepsilon}\,\Omega \tag{D.37}$$

$$(\partial_t + \vec{v}\cdot\nabla)\left[\widehat{\operatorname{dev}\varepsilon_1}\right] \;=\; \operatorname{dev} D_1$$
$$+\Omega\,\widehat{\operatorname{dev}\varepsilon_1} - \widehat{\operatorname{dev}\varepsilon_1}\,\Omega \tag{D.39}$$

Whenever D and ε (or D_1 and ε_1) do not commute, Eqs. (D.37,D.39) cease to be valid.

In Appendix D.2.3 entitled "Simple example", as a result of the restricted validity of Eqs. (D.37,D.39), the two sentences immediately after Eqs. (D.46-D.49) should be amended as follows:

Modified sentence in Appendix D.2.3
Eqs. (D.23,D.24), (D.29,D.30) and (D.46-D.49) are sufficient to describe the material evolution using a closed set of equations.

In Appendix D.2.4 entitled "Complex example", consistently with the remark in Appendix D.2.2, the small deformation notation $\dot{\varepsilon}_{\mathrm{ck}}$ must be replaced with D_{ck}. The notation ε_2 should also be changed to $\varepsilon_{\mathrm{intra}}$. Correspondingly, Eqs. (D.50, D.51) should be amended as follows:

Modified equations in Appendix D.2.4

$$\mathcal{E} = G_1 \, (\widehat{\mathrm{dev}} \, (\varepsilon_1))^2 + G_2 \, (\widehat{\mathrm{dev}} \, (\varepsilon_{\mathrm{intra}}))^2 \tag{D.50}$$

$$\begin{aligned} \mathcal{D} = {} & \eta_{\mathrm{cyto}}(\mathrm{dev}\, D_{\mathrm{intra}} - \mathrm{dev}\, D_1)^2 \\ & + \sigma_{\mathrm{Y}} \, |\mathrm{dev}\, D - \mathrm{dev}\, D_{\mathrm{intra}} - \mathrm{dev}\, D_3 - \mathrm{dev}\, D_{\mathrm{ck}}| \\ & + \eta_3 (\mathrm{dev}\, D_3)^2 + \frac{1}{2}\eta_{\mathrm{sw}}(\mathrm{tr}\, D_{\mathrm{intra}} - r_{\mathrm{sw}} + \mathrm{tr}\, D_{\mathrm{ck}})^2 \\ & + \frac{1}{2}\eta_{\mathrm{apo}}(\mathrm{tr}\, D - \mathrm{tr}\, D_{\mathrm{intra}} - \mathrm{tr}\, D_{\mathrm{ck}} + r_{\mathrm{apo}})^2 \end{aligned} \tag{D.51}$$

In Appendix D.2.4 entitled "Complex example", as a result of the restricted validity of Eqs. (D.37,D.39), the sentence after Eq. (D.54) should be reformulated as follows:

Modified sentence in Appendix D.2.4

Eqs. (14, 22, D.29, D.30, D.50-D.54) close the set of equations which determine the evolution of the tissue.